# Differential Geometry

## Supplementary Examination

1. Verify that the following statements are true or false (without justification) (2+2+2+2+2=10 points).
   (i)$-ydx+xdy$ is a non-vanishing 1 form on $\mathbb{S}^1$.
   (ii)$\mathbb{RP}^2$ is orientable.
   (iii)There exists a 2-form $\alpha$ on $\mathbb{R}^4$ such that $\alpha \wedge \alpha \neq 0$.
   (iv) Mobius Strip $M$ can be written as $M = f^{-1}(c)$, where $f : \mathbb{R}^3 \to \mathbb{R}$ is a smooth function and $c$ is a regular value of $f$.
   (v)$\mathbb{R}^2$ and $\mathbb{S}^1 \times \mathbb{R}$ have same Gauss curvature.

2. (10 points) Let $V$ be a finite dimensional vector space and $f, g : V \to \mathbb{R}$ be two non-zero linear functionals such that $\ker(f) = \ker(g)$. Sow that there exists a non-zero constant $c$ such that $g = cf$.

3. (10 points) Compute $d(f^*\omega)$ and $f^*(d\omega)$ where $f : \mathbb{R} \to \mathbb{R}^2$ with $f(x) = (x, -x)$ and $\omega = dx + dy$. Check that $d(f^*\omega) = f^*(d\omega)$.

4. (10 points) The curve $\gamma : \mathbb{R} \to \mathbb{S}^1 \times \mathbb{R}$ defined by

   $$\gamma(t) = (\cos t, \sin t, t)$$

   is a geodesic in $\mathbb{S}^1 \times \mathbb{R}$.

5. (10 points) Give an example of a 2-form $\omega$ on $\mathbb{R}^3$ such that if $\mathbb{S}^2$ is the unit sphere, the integral $\int_{\mathbb{S}^2} \omega \neq 0$.